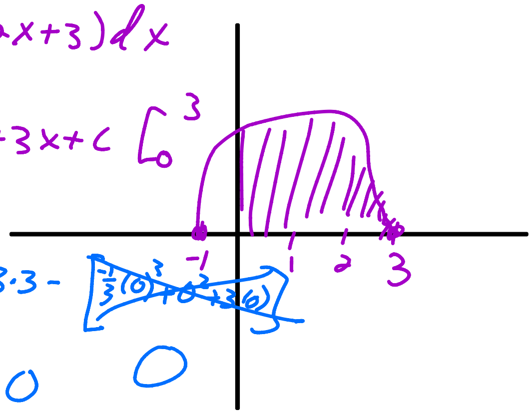
Period 1, Jan 29, 2024

1. The area under the graph of $g(x) = -x^2 + 2x + 3$ from 0 to 3 is ___9___
Show set up and ALL work.

$$\int_0^3 (-x^2+2x+3)\,dx$$

$$-\frac{1}{3}x^3 + x^2 + 3x + c \Big|_0^3$$



$$-\frac{1}{3}(3)^3 + (3)^2 + 3\cdot 3 - \boxed{\cancel{\tfrac{1}{3}(0)^3 + 0^2 + 3(0)}}$$

0

$$-\frac{27}{3} + 9 + 9 - 0$$

$$-9 + 9 + 9 = 9$$

$$\underbrace{-x^2+2x+3}_{-1\cdot 3=-3}$$

$$3 - 1 = 2$$

$$\underbrace{-x^2+3x}\ \underbrace{-x+3}$$

$$x(-x+3) + 1(-x+3)$$

$$(-x+3)(x+1) = 0$$

$x = -1$

$x = 3$

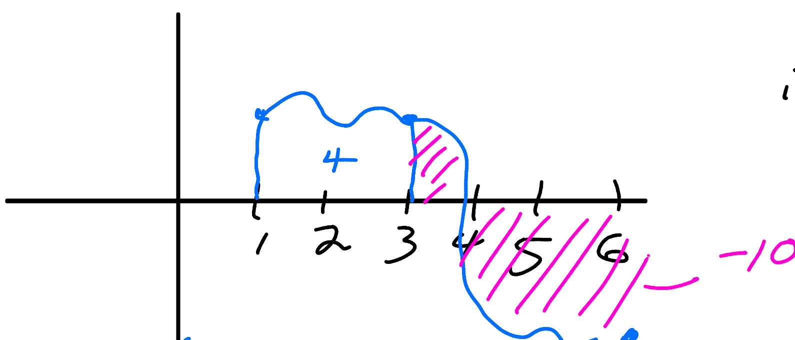
$$\int_3^6 [3F(x)-1]\,dx = 3\int_3^6 F(x)\,dx - \int_3^6 1\,dx = 3\cdot -10 - 3$$

$F(x)$

3. If $\int_1^3 f(x)\,dx = 4$ and $\int_1^6 f(x)\,dx = -6$, then $\int_3^6 [3f(x)-1]\,dx =$ ___-33___
Show the set-up that led to your answer.

$$\int_1^6 F(x)\,dx = \int_1^3 F(x)\,dx + \int_3^6 F(x)\,dx$$

$$-6 = 4 + \int_3^6 F(x)\,dx$$



$$\int_3^6 F(x)\,dx = -10$$

$$\int_3^6 1\,dx = x + c \Big|_3^6 = 3$$

$$6 - 3 = 3$$

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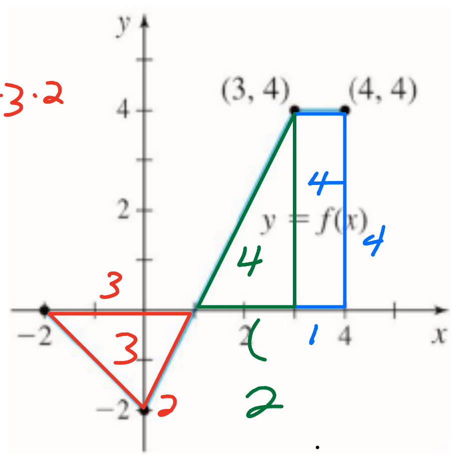
4. The graph of the piecewise function is below. What is $\int_{-2}^{4} f(x)\,dx$?

$\frac{1}{2}\cdot 3\cdot 2$



$\frac{1}{2}\cdot 2\cdot 4 = 4$

$\int_{-2}^{4} F(x)dx = -3+4+4 = 5$

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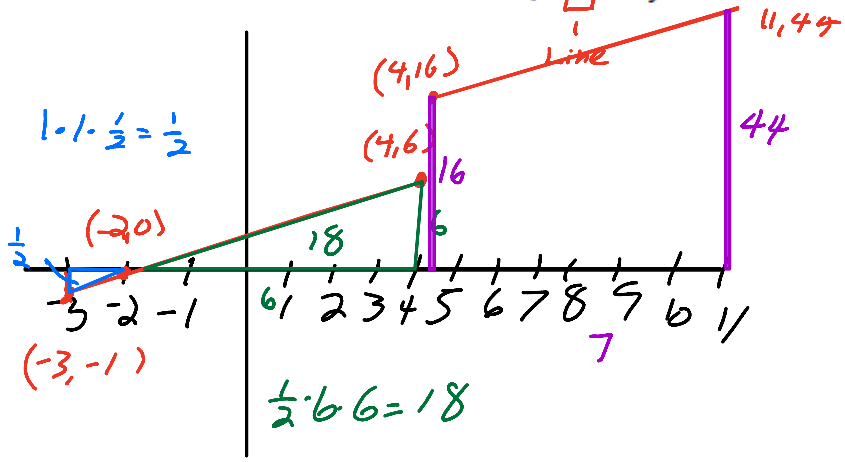
5. Find $\int_{-3}^{11} h(x)\,dx$ where $h(x) = \begin{cases} \overbrace{x+2}^{\text{Line}}, & if\ -3 \le x \le 4 \\ \underbrace{4x}_{\text{Line}}, & if\ 4 < x \le 11 \end{cases}$

$y = x+2$

x	y
-3	-1
-2	0
4	6

$1\cdot 1\cdot \frac{1}{2} = \frac{1}{2}$



$\frac{1}{2}\cdot 6\cdot 6 = 18$

$y = 4x$

x	y
4	16
11	44

$\int_{-3}^{4} h(x)dx = -\frac{1}{2} + 18 = 17\frac{1}{2}$

$\int_{4}^{11} h(x)dx = \frac{1}{2}(16+44)\cdot 7 = \frac{1}{2}\cdot 60\cdot 7$

$30\cdot 7 = 210$

$\int_{-3}^{11} h(x)dx = 17\frac{1}{2} + 210 = 227\frac{1}{2}$

$$\int_{-3}^{4}(x+2)\,dx + \int_{4}^{11} 4x\,dx$$

$$\frac{1}{2}x^2+2x+c\,\Big[_{-3}^{4}$$

$$\left[\frac{1}{2}(4)^2+2(4)\right]-\left[\frac{1}{2}(-3)^2+2(-3)\right]$$

$$\left[8+8\right]-\left[\frac{9}{2}-6\right]$$

$$16-\left(-1\tfrac{1}{2}\right)$$

$$16+1\tfrac{1}{2}=17\tfrac{1}{2}$$

$$2x^2+c\,\Big[_{4}^{11}$$

$$2(11)^2-2(4)^2$$

$$2(121)-2\cdot 16$$

$$242-32=210$$

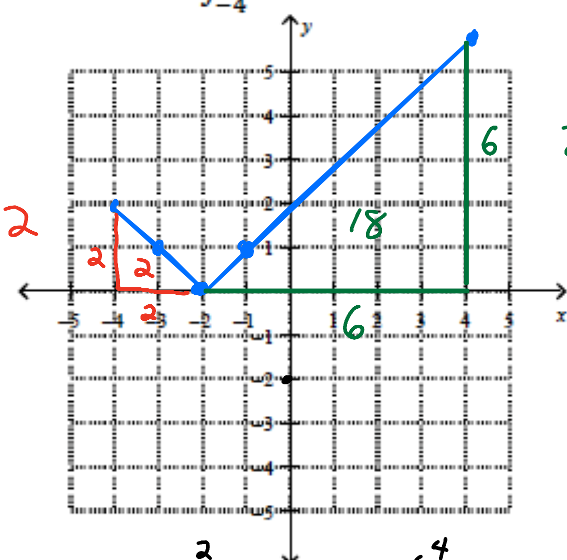
$$17\tfrac{1}{2}+210=227\tfrac{1}{2}$$

$$\int_{-4}^{4}|x+2|\,dx = \underline{2+18=20}$$

$|x+2|=y$

x	y
-2	0
-3	1
-1	1
-4	2
4	6

$\frac{1}{2}\cdot 2\cdot 2=2$



$\frac{1}{2}\cdot 6\cdot 6=18$

$$\int_{-4}^{2}(-x-2)\,dx + \int_{-2}^{4}(x+2)\,dx$$

$$-\frac{1}{2}x^2-2x\,\Big[_{-4}^{-2}$$

$$-\frac{1}{2}(-2)^2-2(-2)-\left[-\frac{1}{2}(-4)^2-2(-4)\right]$$

$$-2+4-\left[\cancel{-8+8}\right]=2$$

$$\frac{1}{2}x^2+2x\,\Big[_{-2}^{4}$$

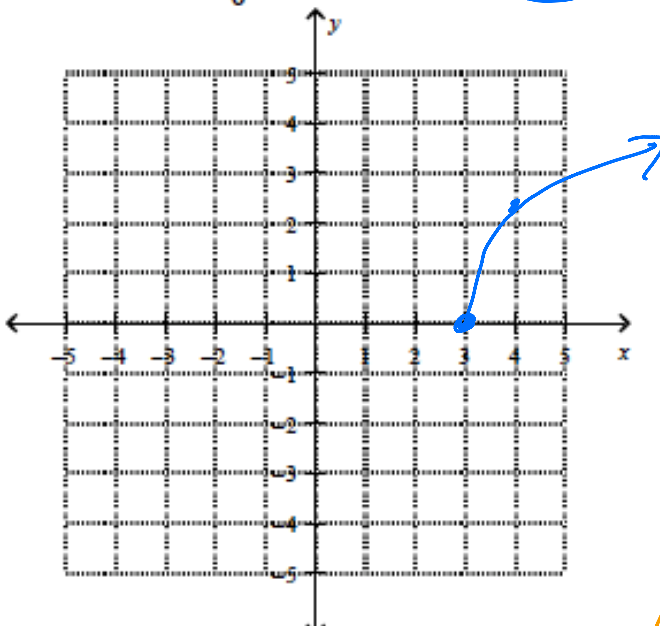
$$\frac{1}{2}(4)^2+2(4)$$

$$-\left[\frac{1}{2}(-2)^2+2(-2)\right]$$

$$8+8-(-2)$$

$$18$$

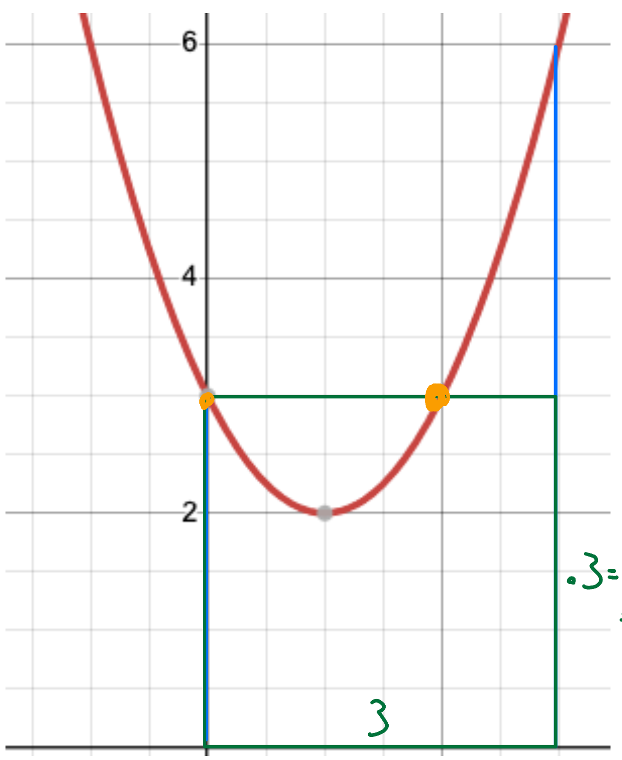
$$\int_0^3 \sqrt{t^2-9}\,dt = 0$$



$$\overbrace{\int_a^b f(x)\,dx}^{9} = \overbrace{f(c)(b-a)}^{3\cdot(3-0)}$$

Area under the Curve

Area of a rectangle with length (b-a)



$y = x^2 - 2x + 3$

$\int_0^3 (x^2 - 2x + 3)\,dx$

$\frac{1}{3}x^3 - x^2 + 3x + c \Big]_0^3$

$\frac{1}{3}(3)^3 - 3^2 + 3\cdot 3 - \left[\cancel{\frac{1}{3}(0)^3 - 0^2 + 3(0)}\right]$

$\frac{27}{3} - 9 + 9 = 9 - 9 + 9 = 9$

$3\cdot 3 = 9$

$\cdot 3 = \text{height}$

$= \text{Average value}$

width $(3-0)$

$3 = f(c)$

$3 = x^2 - 2x + 3$

$-3 \qquad\qquad -3$

$0 = x^2 - 2x$

$0 = x(x-2)$ MVT

$\boxed{x = 0 \quad x = +2 \rightarrow c}$

Example 6: Find the values of c guaranteed by the Mean Value Theorem for Integrals for the function $g(x) = x^3$ over [0, 3].

$$\int_a^b g(x)dx = g(c)(b-a)$$

REC. HEIGHT REC. BASE

Set-up

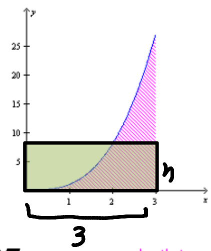
$$\int_0^3 x^3 dx = g(c)(3-0)$$

$$\left.\frac{x^4}{4}\right|_0^3 = g(c)(3)$$

$$\frac{81}{4} = c^3(3)$$

$$g(c) = \frac{27}{4}$$

$g(x) = x^3$



$$c^3 = \frac{27}{4}$$

x-value that gives height of 27/4

$$c = \sqrt[3]{\frac{27}{4}} = \frac{3}{\sqrt[3]{4}} \approx 1.8898$$

$$\frac{h \cdot \cancel{3}}{3} = \frac{81}{4 \cdot 3}$$

$$h = \frac{27}{4}$$

To Find c

$$\sqrt[3]{\frac{27}{4}} = \sqrt[3]{x^3}$$

$$\frac{3}{\sqrt[3]{4}} = c$$

$$\text{Average Value} = \frac{1}{b-a}\int_a^b f(x)\,dx = \frac{\text{Area}}{\text{width}}$$

Average Value

$$\frac{1}{3-0}\int_a^b F(x)dx = h$$

$$\frac{1}{3} \cdot \frac{81}{4} = \frac{27}{4} = h$$

$$\frac{\int_a^b F(x)dx}{b-a} = \frac{\cancel{(b-a)} \cdot F(c)}{\cancel{b-a}}$$

$$\frac{1}{b-a}\int_a^b F(x)dx = F(c) = \text{Average Value}$$

Average velocity From [a To b]

position

$$\int_a^b (v(T)\,dT) = S(b) - S(a) = \text{Area under velocity curve}$$

$$\int_a^b (v(T)\,dT) = \underbrace{[b-a]}_{\text{width}} \cdot V(c)$$

height

"Average Velocity"

$$S(b) - S(a) = [b-a]\cdot V(c)$$

$$\frac{S(b)-S(a)}{b-a} = V(c) = \text{Average velocity}$$

A car's acceleration a in ft/s^2 is measured each second t for $t = 0$ to $t = 8$ and posted in the table.

t	0	1	2	3	4	5	6	7	8
$a(t)$	0	2	4	6	7	7	8	6	2

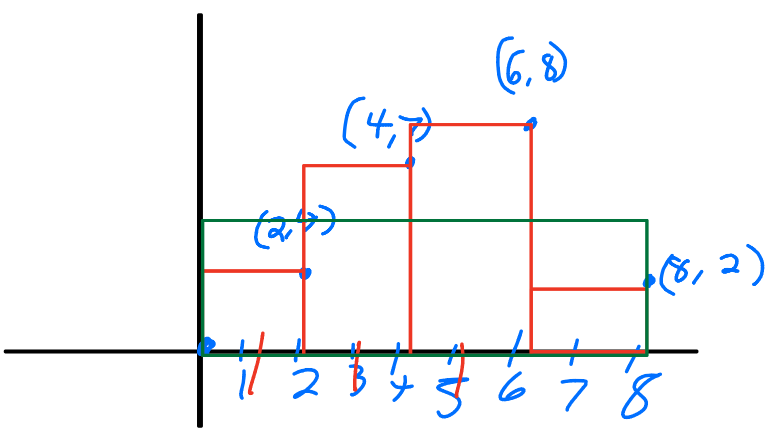
$V(T) = \int a(T)\,dT = 42$

Use a Right Riemann sum with 4 subintervals of equal length to approximate the car's average velocity over the interval from 0 to 8 seconds.

$42 = (8-0)\cdot$ Average Velocity

$$\frac{42}{8} = \frac{8\cdot V}{8}$$

$$5\tfrac{1}{4} = V$$



$2\cdot 4 + 2\cdot 7 + 2\cdot 8 + 2\cdot 2 = 8 + 14 + 16 + 4 = 42$